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HEAT TRANSFER TO A TURBULENT BOUNDARY LAYER WITH VARYING FREE-STREAM VELOCITY AND VARYING SURFACE TEMPERATURE—AN EXPERIMENTAL STUDY

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Abstract—Experimental data are presented for heat transfer to an essentially constant property turbulent boundary layer for various rates of free-stream acceleration. A limited amount of data for free-stream deceleration is also presented. The experimental apparatus was so constructed that surface temperature could be varied in an arbitrary manner, although the bulk of the data presented are for simple steps in surface temperature. It is found that acceleration causes a depression in the heat-transfer rate below what would be predicted assuming a boundary-layer structure such as obtains for constant free-stream velocity. An empirical correlation of the results is presented. When used in conjunction with superposition theory, the results can be used to calculate heat-transfer rates for any arbitrary free-stream velocity variation, and any arbitrary surface temperature variation.

NOMENCLATURE

c_P, specific heat at constant pressure;
f, friction coefficient;
G, free-stream mass velocity, $u_\infty \rho$;
h, convection heat-transfer conductance;
H, absolute humidity;
K, an acceleration parameter, defined by equation (1);
P_s, stagnation pressure;
Pr, Prandtl number;
R, radius of a body of revolution;
Re_x, Reynolds number, defined by equation (7);
$Re_{x-\xi}$, Reynolds number, defined by equation (3a);
Re_m, Momentum thickness Reynolds number, $\delta_m G/\mu$;
St, Stanton number, $h/(Gc_P)$;
$\widetilde{St}$, Stanton number from equations (8) and (6);
T_0, wall surface temperature;
T_∞, free-stream temperature (constant for all tests);
ΔT, convection temperature difference, $T_0 - T_\infty$;
u_∞, free-stream velocity;
x, distance from beginning of plate to point in question;
δ_m, momentum thickness of the boundary layer;
ξ, distance from beginning of plate to a step in surface temperature;
μ, viscosity coefficient;
ρ, fluid density;
ν, kinematic viscosity, μ/ρ;
τ_0, wall shear stress.

INTRODUCTION

INTEREST in the behavior of turbulent boundary layers in a free-stream pressure gradient has, until the last dozen years, been primarily concerned with boundary-layer separation in an adverse pressure gradient (decelerating free-stream). More recent problems associated with the cooling of gas-turbine blades and especially rocket nozzles had led to increased interest in heat transfer under varying free-stream velocity conditions, and in this case the accelerating flow becomes of more practical concern.

Wilson and Pope [1] in 1954 noted that heat-transfer coefficients on the convex side of a gas-turbine blade were considerably lower than anticipated for a turbulent boundary layer, and Wilson [2] in 1957 suggested that acceleration may have caused the boundary layer to return from turbulent to laminar. Sternberg [3], Senoo [4], and Sergienko and Gretsov [5] have observed a similar phenomenon, and more recently Launder [6, 7] has studied this re-transition or "laminarization" phenomenon in considerable detail and has proposed a quantitative criterion for re-transition. Still more recently, Schraub [8] has correlated with the axial pressure gradient the rate at which turbulence "bursts" leave the wall, the results being quite consistent with the observations of Launder.

Definitive heat-transfer data under these conditions are more scarce. But of particular interest are the results of Back, Massier, and Gier [9] for heat transfer from the surface of a supersonic nozzle. Although it is not clear whether or not a complete re-transition occurs, a definite reduction of the heat-transfer coefficient related to the rate of acceleration is observed.

As far as the authors are aware, no attempt has been made to study experimentally heat transfer to a turbulent boundary layer under varying free-stream velocity conditions with surface temperature varying axially in an arbitrary manner. Because the energy equation is linear and homogeneous, the varying surface temperature problem can be handled by superposition if the response of the boundary layer to simple steps in surface temperature is known. An experimental study of the effect of surface temperature steps not only leads to the possibility of a more generally useful solution to the energy equation, but should also provide more information on the influence of free-stream velocity gradients than is obtainable with a completely isothermal surface.

OBJECTIVES

The objectives of this paper are twofold:

(1) To present a representative selection of experimental local heat-transfer data obtained for flow of relatively low-velocity air for accelerating turbulent flows along a flat-plate surface with steps in temperature located at arbitrary points, and a more limited amount of similar data for decelerating flows.

(2) To present a semi-empirical correlation of these data based on a simple solution to the energy equation proposed by Ambrok [10], with empirical modifications to account for surface temperature steps and an apparent effect of pressure gradient on the rate of turbulence production.

It is hoped that the experimental data will be useful to theoreticians interested in accelerating and decelerating turbulent flows, and that the proposed empirical correlation will be useful for engineering analysis, especially under variable surface temperature conditions where superposition can be used to build up solutions for any arbitrary variation of surface temperature.

EXPERIMENTAL APPARATUS

The principal elements of the experimental system are shown on Fig. 1. Air at near atmospheric pressure and temperature was supplied by a 2000 cfm blower, and, after passing through a straightening section and a 60 × 60 mesh screen, entered the system of rectangular nozzles shown at the left-hand side of the diagram. The test

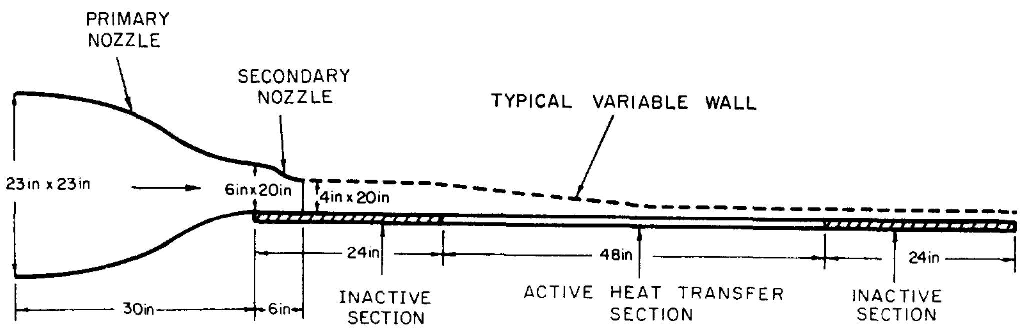

FIG. 1. Diagram of test system.

section consisted of a rectangular duct, one wall ofwhich was flat and was instrumented for heat-transfer measurements (described in more detail below), and the other wall of which could be shaped to provide any desired distribution of free-stream velocity and pressure along the duct. The duct then discharged to the atmosphere.

The lower wall of the duct consisted of a polished $\frac{1}{8}$ in thick copper surface segmented into cells separated by approximately 0·025 in plastic spacers. Each cell was approximately 1 in wide (in the direction of flow). There were 96 such cells, each of which was bonded on the lower side to a Bakelite lamina, which was in turn bonded to a rectangular copper water-tube. In the portion of the plate labeled "active heat-transfer section" in Fig. 1 there were installed thermopile heat-flux transducers within the Bakelite laminae between the $\frac{1}{8}$ in thick copper surface plates and the rectangular water tubes. Thermocouples were then installed in the copper surface plates, so that for each 1in segment heat flux and surface temperature could be readily measured. Furthermore, by varying the water temperature, or by closing off the water flow completely, the surface temperature could be adjusted to any desired distribution in the flow direction, including steps in surface temperature.

Air free-stream stagnation pressure and temperature were established by conventional probes mounted at the outlet of the secondary nozzle. The free-stream velocity and pressure distribution along the duct were established by total pressure probes and static pressure taps in the side walls.

In the tests to be reported free-stream velocity was varied in the range 30 ft/s to 200 ft/s. Stagnation air temperature was typically around 90°F, and the active surface was typically cooled to around 70°F by using straight tap water in the cooling tubes. Thus the heat-transfer temperature differences were small, the Mach number was low, and the test results were then for an essentially constant property boundary layer.

The uncertainty in the local heat-transfer results (specifically the Stanton number) has been estimated to be approximately ± 6 per cent. The largest contributing factor from the instrumentation was the uncertainty in the heat-flux meter responses. The heat meters were calibrated in place using an external heat source, and additional tests were run at constant free-stream velocity so as to check the system against established data. Another important source of uncertainty arises from the deliberate use of small temperature differences. A one or two degree temperature variation over the flow cross section for the fluid coming out of the nozzle can introduce a substantial error. Although the primary nozzle was insulated, some heat transfer could take place, the amount depending partially upon the surrounding ambient temperature. Any heat transfer in the primary nozzle results in some temperature stratification which is difficult to detect. An error of this type results in all of the Stanton numbers for a particular test run being biased in one direction. Thus it is felt that the apparatus was sensitive to relatively small effects when the various test points for any one test run are compared, but that the uncertainty in the absolute value of the Stanton numbers for any one test run is considerably larger. Therefore meaningful conclusions about the absolute value of Stanton number, and comparison of the results with analysis, should be based on a consideration of a number of different test runs, and not any one particular test run.

The boundary layer on the test plate at the exit from the secondary nozzle is believed to have been in all cases laminar, a fact that will be more apparent when the effect of a rapid acceleration is examined later. Accordingly a boundary-layer trip, consisting of a $\frac{1}{8}$ in wide strip of "Scotch Masking Tape", was fastened to the surface at a point where the momentum thickness Reynolds number was in the range 300–500.

Velocity and temperature profiles were measured on the plate under constant, relatively low-velocity conditions, merely as a check on the operation of the system. However, no attempt was made to make such measurements in the highly accelerated flow, as the direct heat-transfer measurements proved to be far more meaningful.

TEST PROGRAM AND PRESENTATION OF RESULTS

The test program consisted of forty-six test runs in which the free-stream velocity was systematically varied to provide various rates of acceleration and deceleration, and the surface temperature was varied to provide steps in temperature difference at various points along the plate, and in one case to provide alternate increases and decreases in temperature difference.

A selected representative set of the results for twelve of the test runs are presented in Figs. 2–13. Sufficient information is presented on each figure so that the results for a single test run can be rather completely analyzed as desired. In the upper panel of each figure the free-stream velocity, u_∞, is plotted, along with a local non-dimensional acceleration parameter,

$$K = \frac{\nu}{u_\infty^2}\frac{du_\infty}{dx} \tag{1}$$

The maximum value of K attained in each test is indicated in the caption. Also indicated in the upper panel is the location of the trip and the value of the length and momentum thickness Reynolds numbers near the end of the constant velocity section. All parameters are plotted as a function of distance along the plate, measured from the start of the secondary nozzle.

The center panel contains the measured values of the local Stanton number. The solid line in the center panel is a plot of the Stanton number predicted by a relatively simple solution to the energy equation to be described later.

The lower panel is a plot of surface temperature. For most of the test runs surface temperature was adjusted nominally for a simple step. However, since it was not feasible to make any of the surface sections completely adiabatic, and there was some heat leak between sections (accounted for in the data reduction procedure), steps were only approximated.

The sequence of test runs shown start with two (Figs. 2 and 3) with constant free-stream

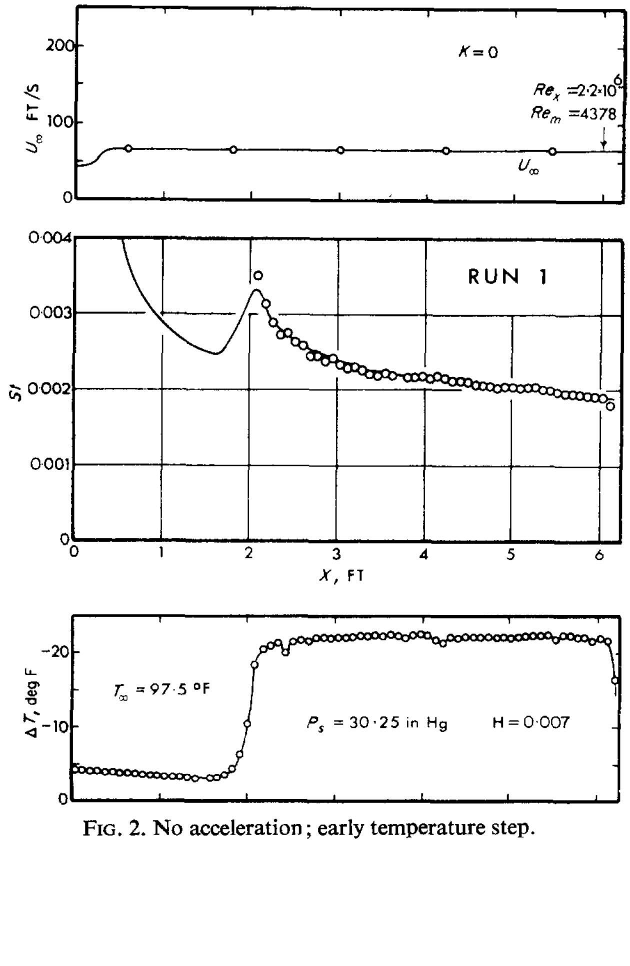

FIG. 2. No acceleration; early temperature step.

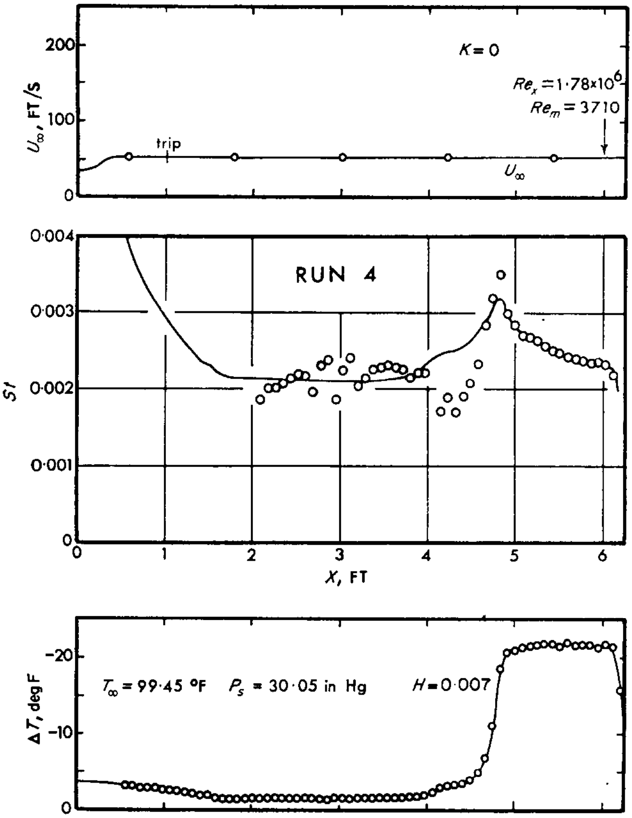

FIG. 3. No acceleration; late temperature step.

velocity ($K = 0$), but with surface temperature steps at two different points. These tests were carried out to check the performance of the apparatus against established data, and will be discussed below. Figures 4 and 5 show the results of a mild acceleration with the surface temperature step at first near the middle of the accelerating region, and finally rather late.

Figures 6 and 7 are similar, but with a much stronger acceleration. Figures 8, 9, and 10 show the effects of successively stronger accelerations with the surface temperature step before the acceleration. Figure 11 is similar to Fig. 10, but with a late acceleration so that the momentum thickness of the boundary layer at the beginning of acceleration is much greater.

Figure 12 shows one case of a decelerating flow, but following an acceleration. Decelerating flows were not investigated extensively because it was not possible to obtain significantly greater negative values of K without encountering the first stages of boundary-layer separation, or stall. In the diffuser section of Fig. 12 the duct height was varied from 1½ to 3 in, but the velocity decreased only thirty-seven per cent because of the increased boundary-layer thickness. Velocity and temperature traverses over the outlet section indicated that the boundary layer had not filled the duct, but nevertheless this possibility placed a limitation on the tests that could be run, and it was not possible to obtain higher negative K by operating at lower velocities.

Finally, Fig. 13 shows the results for an accelerating flow for which the surface temperature was increased and decreased alternately in order to test methods for analytically predicting such behavior.

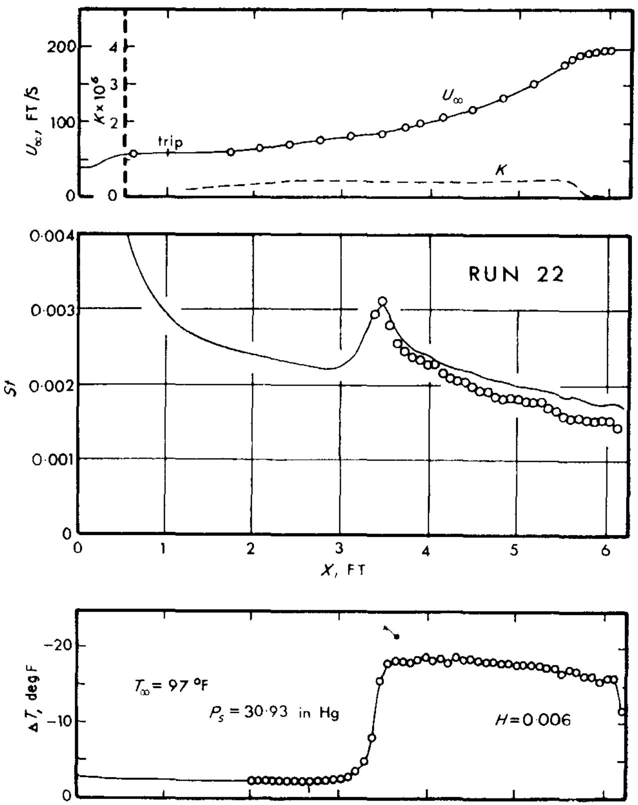

FIG. 4. Acceleration, $K_{max} = 0{\cdot}521 \times 10^{-6}$, early temperature step.

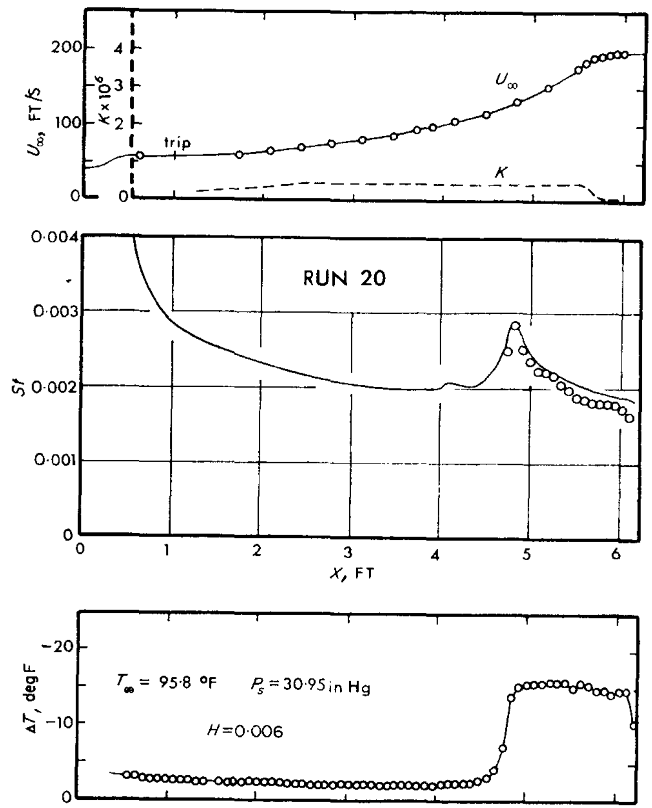

FIG. 5. Acceleration, $K_{max} = 0{\cdot}512 \times 10^{-6}$, very late temperature step.

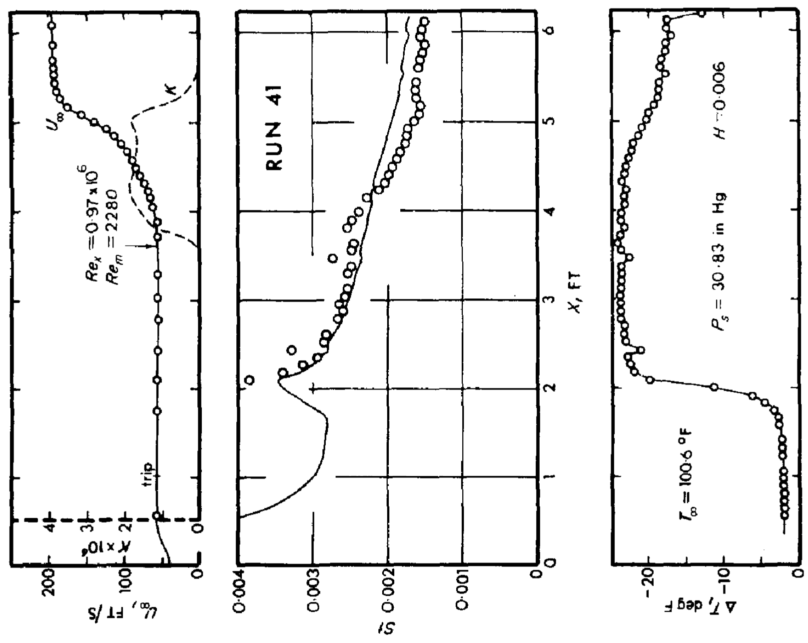

FIG. 6. Acceleration, $K_{max} = 1{\cdot}84 \times 10^{-6}$, very early temperature step.

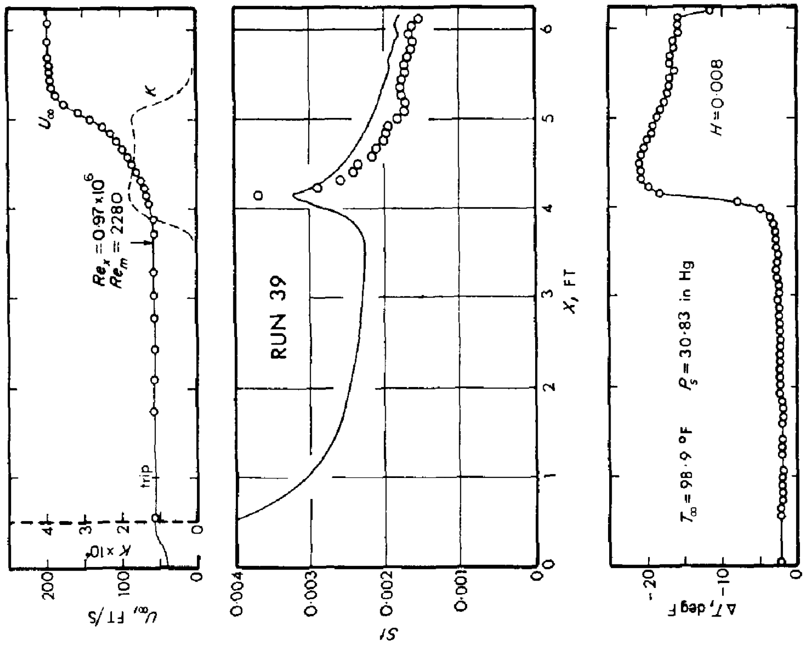

FIG. 7. Acceleration, $K_{max} = 1{\cdot}84 \times 10^{-6}$, late temperature step.

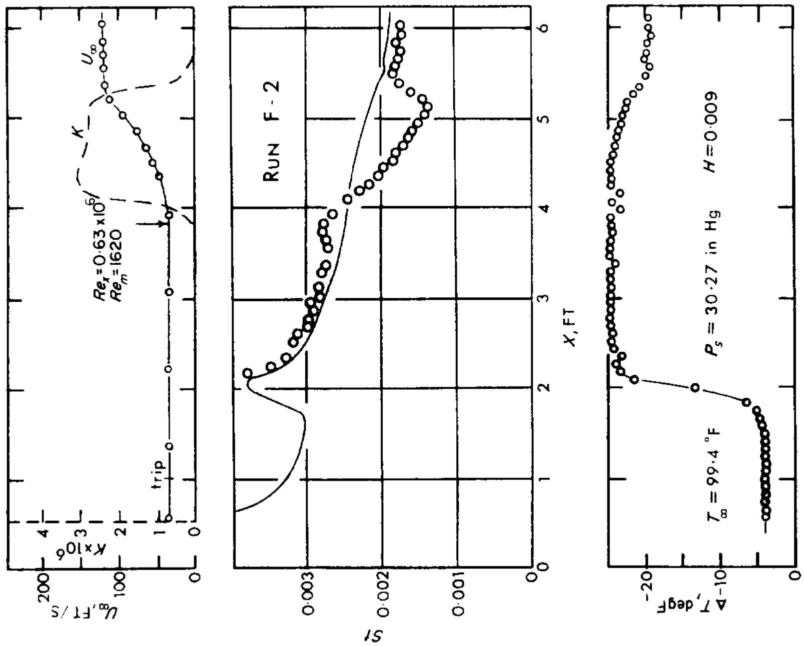

FIG. 9. Acceleration, $K_{max} = 3.04 \times 10^{-6}$, very early temperature step.

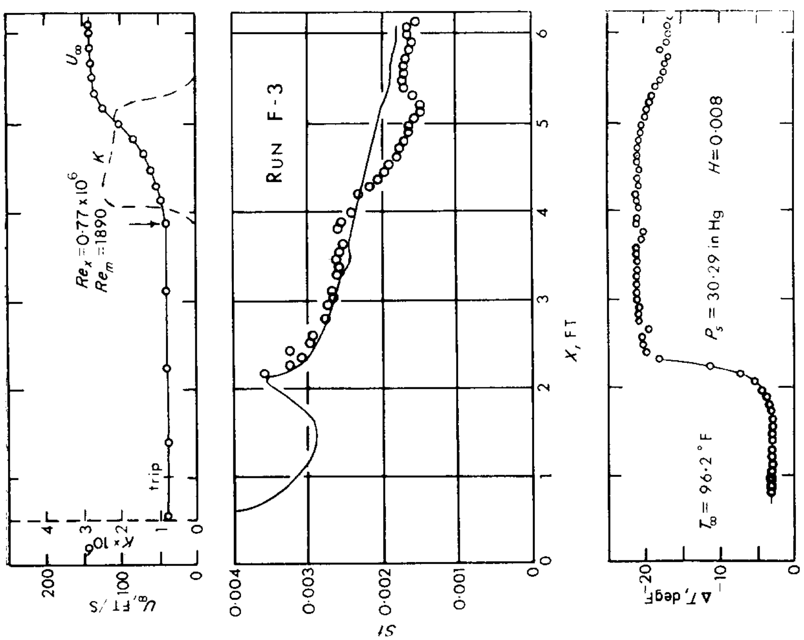

FIG. 8. Acceleration, $K_{max} = 2.52 \times 10^{-6}$, very early temperature step.

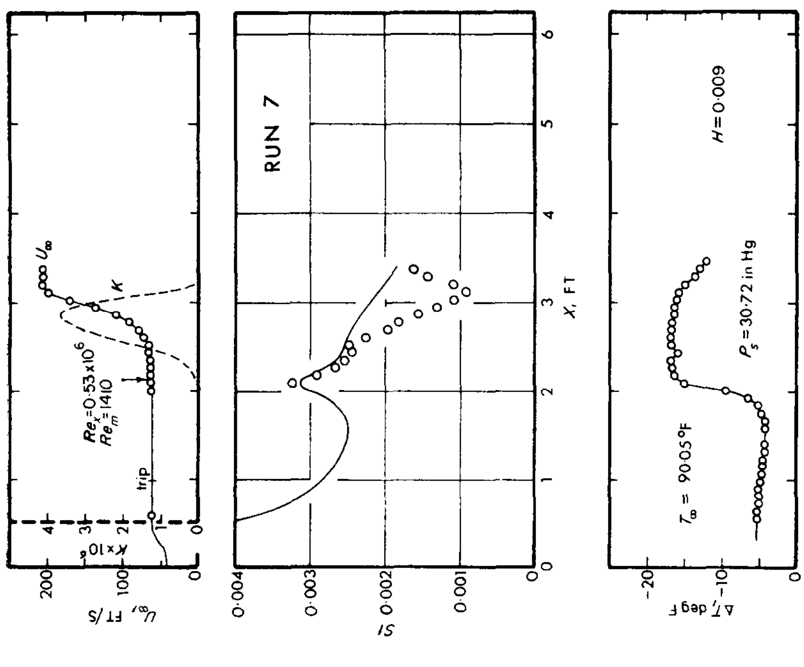

FIG. 10. Acceleration, $K_{max} = 3.51 \times 10^{-6}$, acceleration at low momentum thickness Reynolds number.

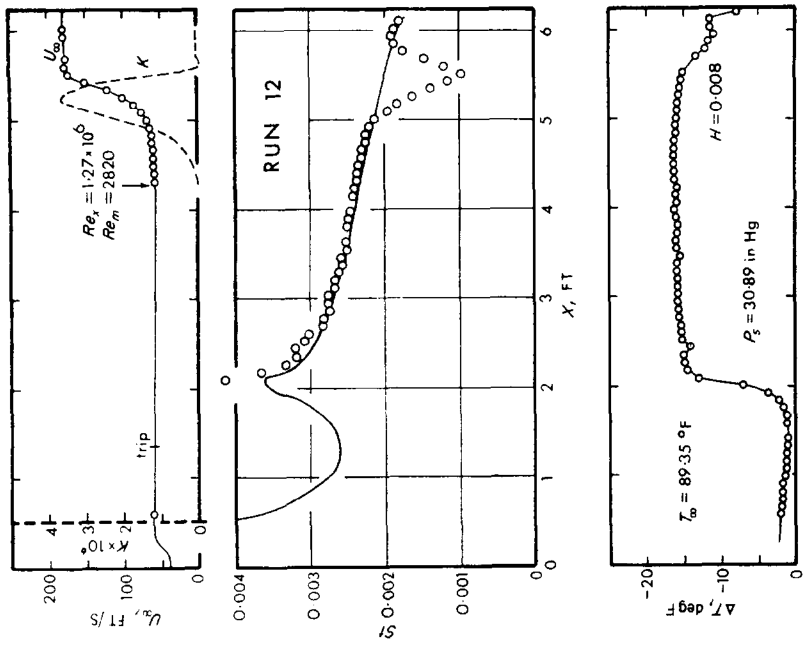

FIG. 11. Acceleration, $K_{max} = 3.39 \times 10^{-6}$, acceleration at high momentum thickness Reynolds number.

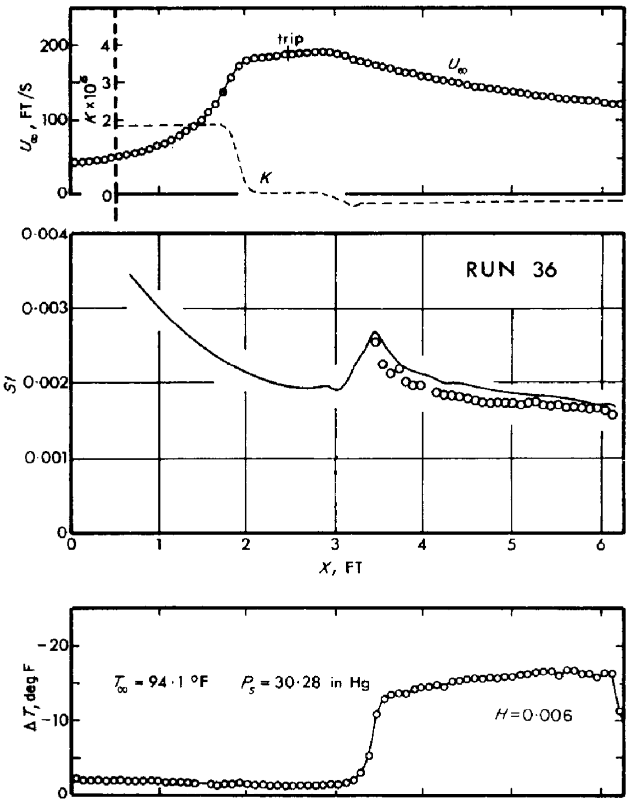

FIG. 12. Deceleration, following an acceleration, late temperature step.

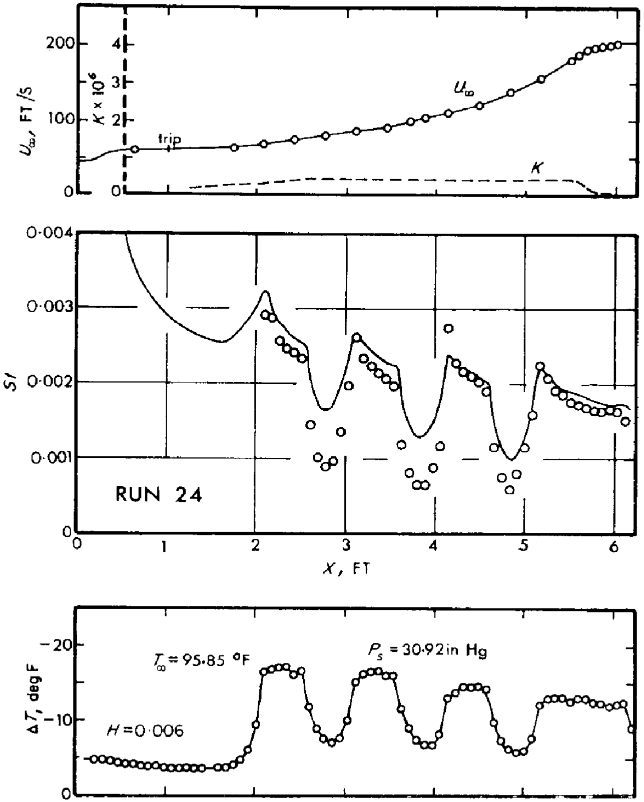

FIG. 13. Acceleration, $K_{max} = 0{\cdot}505 \times 10^{-6}$, alternate steps in surface temperature.

COMPARISON WITH AN ANALYTIC SOLUTION

Experimental data of the type presented is of limited usefulness unless it can be compared against a reasonably general analytic solution, for only then can the results be extrapolated to other operating conditions. Many procedures may be found in the literature for calculating heat transfer to a turbulent boundary layer with varying free-stream velocity and varying surface temperature, and indeed one of the objectives of this test program was to check experimentally the adequacy of some of these procedures.

A frequently preferred procedure has been to first solve the momentum boundary layer, employing the momentum integral equation and an assumed relationship between the surface shear stress and one or more local integral parameters. Then the heat-transfer rate is deduced by either an assumed relation with the shear stress, or by solution of the energy differential equation after bringing in a law-of-the-wall assumption. However, a simpler and more direct procedure, though perhaps theoretically less satisfying, is to solve the energy integral equation directly, after first introducing an assumed relationship between the surface heat flux and the energy thickness of the boundary layer. This is essentially the scheme proposed by Ambrok [10], although it has apparently been proposed independently by others.

Implicit in the Ambrok method is an assumption that the thermal boundary layer develops relatively independently of the momentum boundary layer. Despite the weakness of this assumption, the Ambrok solution has been found to provide a reasonably good approximation for heat transfer to a gas in supersonic nozzles. For example, the experimental data of Back, Massier, and Gier [9], at least for relatively high stagnation pressures, can be quite accurately predicted. For this reason, together with its simplicity and compactness, the Ambrok solution has been chosen as a primary basis for comparison with the present experimental data. The data, of course, are presented in sufficient detail so that other procedures can be compared.

Actually an empirical modification of the Ambrok solution will be used. In its original form (although the coefficient, and the power on the Prandtl number, have been slightly modified) it may be written,

$$St(x) = 0{\cdot}0295\,Pr^{-0\cdot4}\frac{R^{0\cdot25}\,\Delta T^{0\cdot25}\,\mu^{0\cdot20}}{\left(\int_o^x R^{1\cdot25}\Delta T^{1\cdot25}\,G\,\mathrm{d}x\right)^{0\cdot20}} \qquad (2)$$

Equation (2) is presumably applicable for flow over or inside an axi-symmetric body (varying R), and flow with arbitrarily varying surface-to-free-stream temperature difference (varying ΔT). However, more precise procedures are available for calculating the influence of varying ΔT for the case of *constant* G, and it would be desirable to modify equation (2) so that it reduces to a more accurate equation for this case.

Consider the case of $\Delta T = 0$ from the surface origin to ξ, and then $\Delta T =$ *constant* from ξ to x. Then equation (2) may be written,

$$St(\xi, x) = 0{\cdot}0295\,Pr^{-0\cdot4}\,Re_{x-\xi}^{-0\cdot2} \qquad (3)$$

where $$Re_{x-\xi} = \frac{1}{R^{1\cdot25}\mu}\int_\xi^x R^{1\cdot25}\,G\,\mathrm{d}x \qquad (3a)$$

On the other hand, for the case of constant G (and constant R), Reynolds *et al.* [11], among others, have shown that the effect of a simple step in ΔT can be quite adequately represented for a gas by,

$$St(\xi, x) = 0{\cdot}0295 Pr^{-0\cdot4}\,Re_x^{-0\cdot2}\left(1-\left(\frac{\xi}{x}\right)^{\frac{9}{10}}\right)^{-\frac{1}{9}} \qquad (4)$$

where $$Re_x = xG/\mu$$

Equation (4) can be quite well approximated by the simpler form,

$$St(\xi, x) = 0{\cdot}0295 Pr^{-0\cdot4}\,Re_x^{-0\cdot2}\left(1-\frac{\xi}{x}\right)^{-0\cdot12} \qquad (5)$$

But equation (5) can now be rewritten as,

$$\left.\begin{aligned} St(\xi, x) &= 0{\cdot}0295 Pr^{-0\cdot4}\,Re_x^{-0\cdot08}\,Re_{x-\xi}^{-0\cdot12} \\ \text{where} \quad Re_{x-\xi} &= (x-\xi)G/\mu \end{aligned}\right\} \qquad (6)$$

Comparison of equation (3) with equation (6) indicates that the latter includes some influence of the development of the momentum boundary layer prior to the temperature step, whereas the former is completely independent of the history of the boundary layer prior to the step. Since equation (6) corresponds very well with the experimental data of Reynolds *et al.* [11] for *constant* G, it will now be simply suggested that equation (6) may also be a better approximation for the case of variable G and R where there is a step in ΔT. Thus we propose employing equation (6) with $Re_{x-\xi}$ defined by equation (3a), and Re_x defined as,

$$Re_x = \frac{1}{R^{1\cdot 25}\mu}\int_0^x R^{1\cdot 25}\,G\,\mathrm{d}x \qquad (7)$$

With an equation for a step in ΔT, the arbitrarily varying surface temperature problem can now be handled by superposition, since the governing differential energy equation is linear in T. Thus,

$$St(x) = \frac{1}{\Delta T(x)}\int_0^x St(\xi, x)\,\mathrm{d}T_o \qquad (8)$$

where the integral is interpreted in the Stieltjes sense.

Since the test results, Figs. 2–13, all involve a continuously varying surface temperature, rather than simple steps, equation (8), together with equation (6), (3a), and (7), was used to generate the solid line curve in the center panel of each figure. The comparison of this curve with the data points will then be used as a basis for discussion of the apparent effects of acceleration and deceleration, and a basis for empirical correlation of these effects.

It should be added that the double integrations involved are tedious and virtually require machine calculation. For hand calculation, equation (2) is far simpler and differs significantly from the proposed procedure only in the region immediately following rather abrupt changes in ΔT. For a rocket nozzle, for example ΔT usually varies considerably along the surface, but seldom abruptly, and the two procedures will yield close to the same results.

DISCUSSION OF TEST RESULTS

Runs 1 and 4, Figs. 2 and 3, were carried out primarily as a check on the test apparatus for constant free-stream velocity and nominally simple steps in surface temperature. As can be seen the agreement between the test results and the modified Ambrok solution is excellent in the region where the temperature difference is of the order of 20 degF. Where ΔT is small the experimental uncertainty becomes large, and the data points on Fig. 3 in the region $x = 2$ ft to $x = 4\cdot 6$ ft have been plotted on the diagram to illustrate the uncertainty.

Figures 4–11 show the effects of a progressive increase in the rate of acceleration, K, along with the effect of placing nominal steps in ΔT at various points along the plate. The probable experimental uncertainty (± 6 per cent), makes it difficult to draw definitive conclusions from the results of any one test run (see earlier discussion of this problem), but when the results for all of the runs are compared (and these figures are only part of them) the following conclusions seem justified:

(a) In the region immediately following an abrupt increase in ΔT, equation (6) fits the experimental data reasonably well, regardless of whether the step occurs in an accelerating region, or in a region of constant free-stream velocity. In other words, equation (6) appears applicable in an accelerating flow so long as the thermal boundary layer is thin relative to the momentum boundary layer.

(b) With the exception noted under (a), there is an unmistakable tendency of acceleration to decrease Stanton number. At moderate values of K it appears that the boundary layer is still turbulent, but has a lower average turbulence intensity than without acceleration.

(c) At values of K greater than about $3\cdot 3 \times 10^{-6}$, as illustrated by Figs. 10 and 11, the Stanton number decreases sharply and approaches what can be predicted if a purely laminar boundary layer is assumed to originate near the beginning of acceleration. Further increases in K (not shown) yielded virtually identical results, i.e. no further depression of Stanton number.

To obtain a better physical understanding of the hydrodynamics of this phenomena, a series of flow visualization tests were carried out in an accelerating flow in a water table by F. A.

Schraub [8]. Schraub used a dye-trace technique in which he counted the frequency with which large-scale bursts of turbulence leave the wall surface and by way of a curved trajectory move out into the fully turbulent part of the boundary layer, where they break up and start a decay sequence. Schraub found a close correlation between the burst frequency and the acceleration parameter, K. And furthermore, bursts from the wall ceased entirely at about $K = 3{\cdot}5 \times 10^{-6}$.

Since the mechanism whereby turbulent bursts are ejected from the wall is not yet fully understood, the reason why free-stream acceleration (or a negative pressure gradient) suppresses burst ejection is equally obscure. However, the effect of this phenomenon on heat transfer is clear enough for it corresponds rather precisely with the reported heat-transfer results. Where the thermal boundary layer is relatively thin, as in the region immediately following a step in surface temperature, the effect of acceleration is small because the heat-transfer mechanism is primarily molecular conduction in the sublayers. Farther downstream, where the thermal boundary layer has penetrated out into the fully turbulent part of the boundary layer, the overall heat-transfer rate is decreased because the turbulent eddy diffusivity for heat transfer has been decreased due to a lower rate of turbulence production. And of course the complete absence of wall bursts for $K > 3{\cdot}5 \times 10^{6}$ corresponds to the apparent re-transition to a laminar boundary layer noted in the heat-transfer results.

Launder [7] employs the parameter K as a criterion for re-transition, and suggests a critical value of 2×10^{-6}. It could be argued that the data on Figs. 8 and 9, where $K = 2{\cdot}52 \times 10^{-6}$ and $3{\cdot}04 \times 10^{-6}$, respectively, do in fact show a complete re-transition. But since the water table results show a continuous relation between K and burst frequency, it is probably not possible to detect from heat-transfer measurements a precise critical value for K.

Another observation that is consistent with the above mechanism description is that there appears to be a considerable delay or lag in the influence of acceleration on the Stanton number. On Fig. 8, for example, the full effect of acceleration on the Stanton number takes about 1 ft to be established. After the velocity returns to constant, the Stanton number tends to recover, but not fully, and this will be noted for a number of the test runs. The average turbulence intensity in the boundary layer at any point is evidently a function of events that have occurred a considerable distance upstream, and although the data are not sufficiently precise to describe the lag quantitatively, it appears that it may be as much as 100–200 momentum thicknesses.

The possibility of stall made it impossible to vary K significantly during the deceleration tests, and Fig. 12 is quite representative of all of the data obtained. The long zone of strong initial acceleration may, through the lag phenomenon described, have some depressive effect over much of the test section. At any rate, the modified Ambrok solution is reasonably close to the data points, and there does appear to be a tendency for Stanton number to increase after an initial depression.

Figure 13 is a test of how well equation (8) and (6) handle a situation where surface temperature varies in a complex manner in an accelerating flow. Here it is apparent that acceleration has the same effect of depressing Stanton number as noted before, for otherwise the analysis follows the data rather satisfactorily. The test data dip somewhat lower than might be anticipated, but the heat flux is so low in the dip region that the experimental uncertainty is considerably greater than the ±6 per cent previously quoted.

A CORRELATION OF THE EFFECTS OF ACCELERATION

The preceding results suggest that it may be feasible to correlate empirically the depression of Stanton number caused by acceleration with some suitable acceleration parameter. The parameter, K, has been used for convenience in describing the test results because it is constructed of easily measurable local quantities, but there is no theoretical basis for assuming that K is the significant parameter. In fact it would seem plausible that boundary-layer thickness should be involved in some way. (On the other hand it is interesting to note that on Figs. 10 and

11, where it is believed that turbulence production was completely inhibited, the behavior is almost identical despite a two-to-one difference in initial momentum thickness Reynolds number.)

A correlation of the measured Stanton number depression [below equation (6)] with K alone is evidently not possible, at least for a non-isothermal surface, because when the thermal boundary layer is thin the effect becomes negligible. Run 22, Fig. 4, perhaps illustrates this fact most clearly. A simple expedient that will yield a reasonable correlation is to divide K by the measured Stanton number. Thus when the thermal boundary layer is relatively thin and Stanton number is high, as in the region near a surface temperature step, the acceleration parameter becomes small.

On Fig. 14 the ratio of measured Stanton number to Stanton number evaluated from equation (6), $St/\widetilde{St}$ is plotted as a function of K/St. The plotted points have been selected from the test data in a sufficiently random fashion so as to include data both near an abrupt increase in ΔT, and well downstream of such abrupt changes. To avoid including the effect of the lag discussed above, the correlation points were chosen where K had been approximately constant for a distance of at least 100–200 momentum thicknesses. The plotted data include additional test runs not contained in this paper, and include the deceleration data.

The results indicate a definite trend that can be approximated within the experimental uncertainty of the data by the simple linear equation,

$$\frac{St}{\widetilde{St}} = 1 - 165\left(\frac{K}{St}\right) \qquad (9)$$

It is difficult to justify a correlation based on K/St on any theoretical grounds because the mechanism responsible for the observed phenomena is simply not understood. Attempts have been made to derive a significant parameter from theoretical considerations [7, 9], leading to K divided by a power of the friction coefficient, but the basis is sufficiently flimsy that equation (9) should for the present be considered as only a purely experimental correlation. As such it should very definitely not be relied upon for decelerating flows that are anywhere close to separation, for one would certainly not anticipate that a simple correlation of this type would be adequate as the wall shear stress approaches zero.

CONCLUSIONS

The following is a summary of the conclusions that may be drawn from this investigation:

(1) Free-stream acceleration (negative pressure gradient) suppresses the generation of turbulence in a turbulent boundary

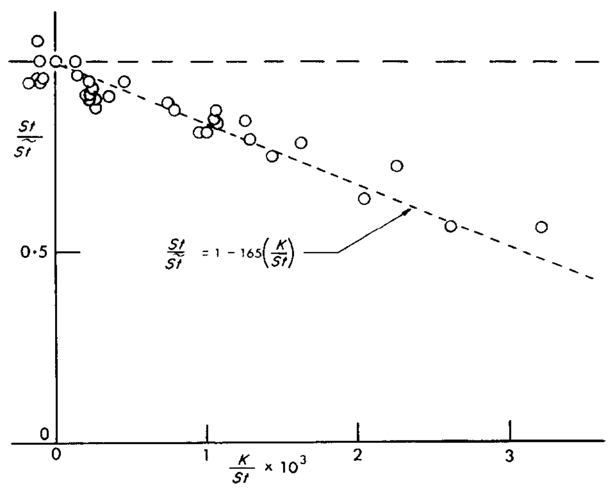

FIG. 14. Correlation of the influence of acceleration.

layer, and causes a decrease in the heat-transfer rate relative to what could be predicted assuming a boundary-layer structure similar to that obtained for no acceleration.

(2) The influence of acceleration on Stanton number can be quantitatively approximated by equation (9). Thus a complete empirical formulation of the test results for a step change in surface temperature under varying free-stream velocity conditions for a gas is,

$$St(\xi, x) = 0{\cdot}0295\, Pr^{-0{\cdot}4}\, Re_x^{-0{\cdot}08}\, Re_{x-\xi}^{-0{\cdot}12} \left(1 - 165\frac{K}{St}\right) \quad (10)$$

where Re_x is defined by equation (7), $Re_{x-\xi}$ is defined by equation (3a), and K is defined by equation (1). This formulation is based on K being relatively constant for some distance before the point of application. Where K is varying, somewhat different behavior may be obtained, and it is probably reasonable to evaluate K at least 100–200 momentum thicknesses upstream.

(3) Equation (10) can be used as an adequate basis for calculating heat-transfer coefficients when surface temperature varies in any arbitrary manner, using superposition, i.e. equation (8). Alternatively equation (2), corrected by equation (9), would appear to be a reasonable approximation where surface temperature does not vary abruptly, and has the advantage of permitting a much simpler computing procedure.

(4) When $K \geqslant 3{\cdot}5 \times 10^{-6}$ turbulence generation is apparently completely inhibited, and, after the residual turbulence has decayed, the boundary layer becomes effectively a laminar one.

Finally it should be noted that while the various equations have been presented in a form applicable to flow over or inside an axi-symmetric body, the test work was entirely carried out in a two-dimensional channel. It is by no means obvious that the same acceleration parameter is applicable to an axi-symmetric flow.

The complete set of experimental data upon which this paper is based may be found in reference 12.

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Résumé—On présente des résultats expérimentaux pour le transport de chaleur à une couche limite turbulente à propriétés essentiellement constantes pour différentes valeurs de l'accélération de l'écoulement libre. Une quantité limitée de résultats pour une décélération de l'écoulement libre est aussi présentée. L'appareillage expérimental était construit de telle façon que la température pariétale pouvait varier de façon arbitraire, bien que l'ensemble des résultats présentés correspondent à de simples sauts de température pariétale. On trouve que l'accélération produit une diminution du flux de transport de chaleur en-dessous de ce qu'on pourrait prévoir en supposant une structure de couche limite telle qu'on obtient pour une vitesse constante de l'écoulement libre. Une corrélation empirique des résultats est présentée. Lorsqu'on les combine par la théorie de la superposition, les résultats peuvent être employés pour calculer les flux de transport de chaleur pour n'importe quelle variation arbitraire de vitesse de l'écoulement libre, et n'importe quelle variation arbitraire de température pariétale.

Zusammenfassung—Für den Wärmeübergang an eine turbulente Grenzschicht mit im wesentlichen konstanten Stoffgrössen werden Versuchswerte bei verschiedenen Beschleunigungswerten des Freistroms angegeben. Ebenso wird ein begrenzter Betrag von Werten für die Verzögerung des Freistromes angeführt. Die Versuchsapparatur war so konstruiert, dass die Oberflächentemperatur beliebig variiert werden konnte, wenn auch die meisten der angeführten Daten für einfache Oberflächentemperaturanstiege gelten. Man findet, dass die Beschleunigung einen Abfall der Wärmeübergangsgeschwindigkeit unter den Wert verursacht, den man bei Annahme einer Grenzschichtstruktur für konstante Freistromgeschwindigkeit vorhersagen würde. Eine empirische Wechselbeziehung der Ergebnisse wird angegeben. Wenn die Ergebnisse in Verbindung mit der Superpositionstheorie gebraucht werden, können sie für die Berechnung der Wärmeübergangsgeschwindigkeiten für jede beliebige Variation von Freistromgeschwindigkeit und Oberflächentemperatur verwendet werden.

Аннотация—Приводятся экспериментальные данные по теплообмену в турбулентном пограничном слое с существенно постоянными свойствами для различных значений ускорения свободного потока, а также некоторые данные для случая замедления свободного потока. Экспериментальная установка была сконструирована таким образом, чтобы можно было произвольно менять температуру поверхности; однако, основное количество данных приводится для определенных значений температуры поверхности. Найдено, что ускорение вызывает уменьшение интенсивности теплообмена ниже того значения, которое можно получить, предположив, что структура пограничного слоя остается такой же, как в случае постоянной скорости свободного потока. Приводится эмпирическое обобщение результатов. В совокупности с теорией наложения результаты можно использовать для расчета интенсивности теплообмена для любых произвольных изменений скорости свободного потока и температуры поверхности.